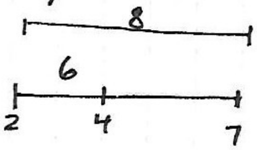
AP Calculus BC

The Definite Integral

1) $\int_2^4 f(x)dx = 6$, $\int_2^7 f(x)dx = 8$, $\int_2^7 g(x)dx = 8$

$\int_4^4 f(x)dx = 0$ $\quad \int_7^2 g(x)dx = -8$ $\quad \int_2^7 9\, g(x)dx = 72$

$\int_4^7 f(x)dx \rightarrow$



$\int_2^7 [g(x) - f(x)]\,dx = 0$

$\boxed{2}$

$\int_2^7 [5g(x) - f(x)] = 5\int_2^7 g(x)dx - \int_2^7 f(x)dx = 32$

2) $\int_2^6 \frac{4}{x}dx$

$4\ln|x| + C \Big|_2^6$

$\boxed{4\ln 6 - 4\ln 2}$

3) $\int_0^5 \sqrt{y+4}\,dy$

$\int_0^5 (y+4)^{1/2}dy$

$\frac{2}{3}(y+4)^{3/2} + C \Big|_0^5$

$\frac{2}{3}(9)^{3/2} - \frac{2}{3}(4)^{3/2}$

$18 - \frac{16}{3}$

4) $\int_0^1 (t^5+4t)^{1/2}(5t^4+4)dt$

$\frac{2}{3}(t^5+4t)^{3/2} + C \Big|_0^1$

$\boxed{\frac{2}{3}(5)^{3/2} - 0}$

check:

$\frac{3}{2}(t^5+4t)^{1/2}(5t^4+4)$

5) $\int_3^7 \frac{18x\,dx}{9x^2+5} = \int_3^7 18x(9x^2+5)^{-1}dx$

$\ln(9x^2+5) + C \Big|_3^7$

$\ln(9(49)+5) - \ln(86)$

6) $\int_0^{\pi/4} \sin x\,dx$

$-\cos x + C \Big|_0^{\pi/4}$

$-\frac{\sqrt{2}}{2} - (-1)$

$\boxed{-\frac{\sqrt{2}}{2} + 1}$

7) $\int_0^{\pi/4} 4\sin(4t)\left(8-\cos(4t)\right)^{-1}dt$

$\ln|8-\cos(4t)| + C \Big|_0^{\pi/4}$

$\ln|8+1| - \ln|8-1|$

$\ln 9 - \ln 7 = \ln\frac{9}{7}$

8) $\int_0^{\pi/8} 5^{\cos 4t} \sin 4t\, dt$

$u = \cos 4t$

$du = -4 \sin 4t\, dt$

$-\frac{1}{4}\, du = \sin 4t\, dt$

$u(0) = 1$

$u(\pi/8) = 0$

$-\frac{1}{4}\int_1^0 5^u\, du$

$-\frac{1}{4}\left[\frac{5^u}{\ln 5}\right]_1^0$

$-\frac{1}{4}\left[\frac{5^0}{\ln 5} - \frac{5}{\ln 5}\right]$

$\boxed{\frac{1}{\ln 5}}$

9) $\int_0^4 12x^2 e^{x^3} dx$

$u = x^3$

$du = 3x^2\, dx$

$4\, du = 12x^2 dx$

$u(0) = 0$

$u(4) = 64$

$4\int_0^{64} e^u\, du$

$4\left[e^u\right]_0^{64}$

$4\left[e^{64} - 1\right]$

10) $\int_0^{4/7} \frac{dx}{49x^2+16}$

$\int_0^{4/7} \frac{dx}{16+49x^2} \rightarrow \int_0^{4/7} \frac{dx}{16\left(1+\frac{49x^2}{16}\right)} \rightarrow \int_0^{4/7} \frac{dx}{16\left(1+\left(\frac{7}{4}x\right)\right)^2}$

$\frac{1}{28}\arctan\left(\frac{7}{4}x\right) dx \Big|_0^{4/7}$

$\frac{1}{28}\arctan(1) - \frac{1}{28}\arctan 0$

$\frac{1}{28} \cdot \frac{\pi}{4}$